

# THE TRANSITION OF VIRTUAL PHOTONS INTO PSEUDOSCALAR MESONS

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The possibility to constrain the meson distribution amplitude from  $\gamma^* \gamma^* \rightarrow \pi, \eta, \eta'$  transitions is investigated. It is shown that for a large range in the two photon virtualities the transition form factors are essentially independent of the distribution amplitudes. This in turn entails parameter-free predictions of QCD.

Since the advent of the CLEO measurement <sup>1</sup> of the  $\gamma^* - P$  transition form factor ( $P = \pi, \eta, \eta'$ ) for quasi-real photons many papers appeared that have been devoted to the theoretical analysis of these form factors. It became evident from these analyses of the CLEO data that the distribution amplitudes for pseudoscalar mesons are close to the asymptotic form,  $\Phi_{AS}(\xi) = 3(1 - \xi^2)/2$ , where  $\xi = 2x - 1$ , and  $x$  is the usual momentum fraction carried by the quark inside the meson. This result, although not very precise, had a strong impact on the phenomenology of hard exclusive reactions. Thus, for instance, earlier conjectures of large contributions from soft physics to the pion's electromagnetic form factor or to two-photon annihilations into pairs of pseudoscalar mesons became substantiated. These contributions, although formally representing power corrections to the asymptotically leading twist ones, seem to dominate for momentum transfers of the order of a few GeV. The meson distribution amplitudes are also an input for the calculation of charmonium or  $B$ -meson decays into pairs of pseudoscalar mesons. A good knowledge of the distribution amplitudes would enhance prospects of extracting information on CP violations from the latter process.

In this talk I am going to report on a recent paper by M. Diehl, C. Vogt and myself <sup>2</sup> where we investigated what information on the distribution amplitudes can be extracted from  $\gamma^* \gamma^* \rightarrow P$  transitions beyond that what has been obtained from the CLEO data in the real-photon limit.

Let me begin with the discussion of the  $\gamma^* \gamma^* \rightarrow \pi$  transitions. To leading twist accuracy the transition form factor  $F_{\pi\gamma^*}$  reads <sup>3,4</sup>

$$F_{\pi\gamma^*}(\overline{Q}, \omega) = \frac{f_\pi}{3\sqrt{2}\overline{Q}^2} \int_{-1}^1 d\xi \frac{\Phi_\pi(\xi, \mu_F)}{1 - \xi^2 \omega^2} \left[ 1 + \frac{\alpha_s(\mu_R)}{\pi} \mathcal{K}(\omega, \xi, \overline{Q}/\mu_F) \right], \quad (1)$$

where  $\overline{Q}^2 = (Q^2 + Q'^2)/2$  and  $\omega = (Q^2 - Q'^2)/(Q^2 + Q'^2)$ .  $Q^2$  and  $Q'^2$  denote the (space-like) virtualities of the photons. The hard scattering kernel has

been calculated to next-to-leading order (NLO); the expression for  $\mathcal{K}$  can be found in <sup>2,3,4</sup>. The factorization,  $\mu_F$ , and renormalization,  $\mu_R$ , scales, are chosen to be equal to  $\overline{Q}$  here.  $f_\pi \approx 131 \text{ MeV}$  is the pion decay constant. The pion distribution amplitude,  $\Phi_\pi$ , can be expanded <sup>5</sup> upon Gegenbauer polynomials  $C_n^{3/2}(\xi)$ ,  $\Phi_\pi = \Phi_{\text{AS}} [1 + \sum B_n(\mu_F) C_n^{3/2}(\xi)]$ . The Gegenbauer coefficients,  $B_n$ , which encode the soft physics information required in the calculation of the form factor, evolve with the scale  $\mu_F$ . Using the expansion of the distribution amplitude, the integrals in (1) can be worked out analytically order by order in  $n$ . This results in (for details see <sup>2</sup>)

$$F_{\pi\gamma^*}(\overline{Q}, \omega) = \frac{f_\pi}{\sqrt{2} \overline{Q}^2} \left[ c_0(\omega, \mu_R) + \sum_{n=2,4,\dots} c_n(\omega, \mu_R, \overline{Q}/\mu_F) B_n(\mu_F) \right]. \quad (2)$$

The coefficients  $c_n$  have the remarkable properties

$$c_n \longrightarrow 1 + \frac{\alpha_s}{\pi} \mathcal{K}_n \quad \text{for } \omega \rightarrow 1, \quad c_n \propto \omega^n \quad \text{for } \omega \rightarrow 0. \quad (3)$$

From (3) it is obvious that, in the real photon limit, the transition form factor is  $\propto 1 + \sum B_n$  to LO. To NLO the sum  $\sum B_n$  is slightly resolved due to the running of  $\alpha_s$  and evolution. In practice the analyses of  $F_{\pi\gamma}$  are performed with a truncation of the Gegenbauer series. The simplest analysis assumes  $B_n = 0$  for  $n \geq 4$  <sup>6</sup>. A fit to the CLEO data <sup>1</sup> above  $Q_{\min}^2 = 2 \text{ GeV}^2$  then provides  $B_2(1 \text{ GeV}) = -0.06 \pm 0.03$  to NLO accuracy in the  $\overline{MS}$  scheme.

If one allows for  $B_2$  and  $B_4$  in the analysis there is no unique result for the individual coefficients. Rather there is a strong linear correlation between both the coefficients; only extreme values of  $|B_2|$  and  $|B_4|$ , say above 1 or 2, are ruled out. A compact way of presenting the result of this fit is to quote the values of the linear combinations  $B_2 + B_4$  and  $B_2 - B_4$ , which have approximately uncorrelated errors:  $B_2 + B_4 = -0.06 \pm 0.08$  and  $B_2 - B_4 = 0.0 \pm 0.9$  at a scale of 1 GeV. This illustrates that, within a leading twist NLO analysis, the CLEO data on the  $\gamma^*\gamma \rightarrow \pi$  form factor approximately fixes only the sum  $\sum B_n$  to be close to zero.

Besides the uncertainties due to the choice of  $\mu_F$ ,  $\mu_R$  and  $Q_{\min}$  there is another important one in the analysis of the form factor data that arises from possible power corrections. While our analysis reveals that logarithmic effects suffice to describe the residual  $Q^2$  dependence of the CLEO data for  $Q^2 F_{\pi\gamma}$  above  $2 \text{ GeV}^2$ , substantial power corrections cannot be excluded since it is very difficult to distinguish a power from a logarithmic behaviour in  $Q^2$  with data in the range between 2 and 8  $\text{GeV}^2$ . It is to be emphasized that any estimate of power corrections is subject to a strong model dependence. Leaving this out of consideration, one may arrive at misleading results.

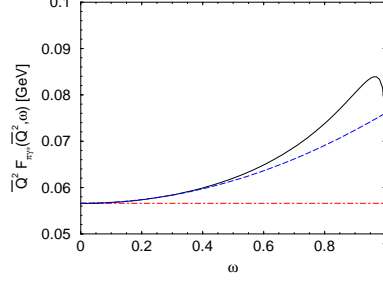


Figure 1. Comparison of the full result (1) for  $\bar{Q}^2 F_{\pi\gamma^*}$  (solid line) with (4) (dashed line) and the  $\omega \rightarrow 0$  limit (dash-dotted line). The form factor is evaluated at  $\bar{Q} = 2$  GeV for the distribution amplitude with  $B_2 = 0.54$ ,  $B_4 = -0.40$ ,  $B_6 = -0.20$  at a scale of 1 GeV.

Let me now turn to the case of two virtual photons. From (3) one sees that for small  $\omega$  a Gegenbauer coefficient  $B_n$  is suppressed in  $F_{\pi\gamma^*}$  by a power  $\omega^n$ . Thus, for small  $\omega$ , one has

$$F_{\pi\gamma^*}(\bar{Q}, \omega) \simeq \frac{\sqrt{2}f_\pi}{3\bar{Q}^2} \left\{ 1 - \frac{\alpha_s}{\pi} + \frac{\omega^2}{5} \left[ 1 - \frac{5}{3} \frac{\alpha_s}{\pi} + \frac{12}{7} B_2 \left( 1 + \frac{5}{12} \frac{\alpha_s}{\pi} \right) \right] \right\} \quad (4)$$

The limiting behavior for  $\omega \rightarrow 0$  has already been given in <sup>3</sup>.

Given the small numerical coefficients in front of  $\omega^2$ , the  $\omega$  independent term in Eq. (4) dominates over a rather large range of  $\omega$ . Even at  $\omega \simeq 0.6$  the  $\omega^2$  corrections amount to less than 15% if  $|B_2| < 0.5$ . Thus, for a wide range of  $\omega$  the  $\gamma^* - \pi$  transition form factor is essentially independent of the pion distribution amplitude. To illustrate the quality of the small- $\omega$  approximations we compare in Fig. 1 the full result (1) for  $F_{\pi\gamma^*}$  with the expression (4) for an extreme example of a distribution amplitude. The full calculation is in agreement with the CLEO data for  $\omega \rightarrow 1$ . We see that, although  $B_2$  in our example is quite large and positive, both approximations are indeed very good for  $\omega \lesssim 0.6$ . Only for  $\omega$ -values near 1 the form factor is sensitive to details of the distribution amplitude. One thus has a parameter-free prediction of QCD to leading-twist accuracy. Any observed deviation from the limiting behaviour for  $\omega \rightarrow 0$  beyond what can reasonably be ascribed to  $\mathcal{O}(\alpha_s^2)$  terms would be an unambiguous signal for power corrections. For small  $\omega$ , the limiting behaviour of the form factor has a status comparable to the famous expression of the cross section ratio  $R = \sigma(e^+e^- \rightarrow \text{hadrons})/\sigma(e^+e^- \rightarrow \mu^+\mu^-)$ .

The  $\gamma^* - \eta$  and  $\gamma^* - \eta'$  transition form factors can be analyzed along the same lines as for the pion. The only complication is that, to order  $\alpha_s$ , there is a contribution from the two-gluon Fock state, its distribution amplitude

mixes with the SU(3)-singlet distribution amplitude under evolution. It has been shown <sup>7</sup> that, in the real photon limit, the CLEO <sup>1</sup> and L3 <sup>8</sup> data on the  $\gamma - \eta^{(\prime)}$  form factors are consistent with approximately equal distribution amplitudes for the  $\pi$ ,  $\eta$  and  $\eta'$  and correspondingly vanishing gluon ones.

For small  $\omega$  one obtains in analogy to (4)

$$F_{P\gamma^*}(\overline{Q}, \omega) = \frac{\sqrt{2} f_P^{\text{eff}}}{3 \overline{Q}^2} \left[ 1 - \frac{\alpha_s}{\pi} \right] + \mathcal{O}(\omega^2, \alpha_s^2). \quad (5)$$

where  $f_P^{\text{eff}}$  are effective, process-dependent decay constants. Using for instance the quark-flavor mixing scheme <sup>9</sup>, one finds for the decay constants  $f_\eta^{\text{eff}} = 0.98 f_\pi$  and  $f_{\eta'}^{\text{eff}} = 1.62 f_\pi$ . At small  $\omega$  and large enough  $\overline{Q}^2$  the ratio of the  $\gamma^* - \eta$ ,  $\eta'$  form factors constitutes an accurate measure of the effective decay constants. This can be used for a severe test of the  $\eta - \eta'$  mixing scheme.

In summary: In the real photon limit the transition form factors essentially provide information on  $\sum B_n$  and these sums seem to be small. Data at large  $Q^2$  are needed in order to determine the size of power corrections. For  $\omega \lesssim 0.6$ , on the other hand, the form factors are essentially independent of the distribution amplitudes. One thus has a parameter-free QCD prediction which well deserves experimental verification. Rate estimates for the running  $B$ -factories reveal that  $F_{\pi\gamma^*}$  should be measurable for  $\overline{Q}^2 \lesssim 4 \text{ GeV}^2$  (for a luminosity of  $30 \text{ fb}^{-1}$  per year).

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